سری هشتم

1) $V_C = \omega_{CD} r_2 \qquad \omega_A = \frac{V_C}{r_1} \qquad V_R = \omega_A \times 2r_1$

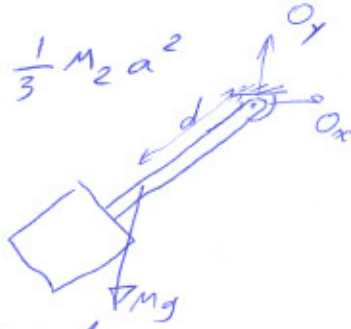
$$\omega_R = \frac{V_R}{r_1 + r_2}$$

2) از مرکز آنی دوران $\Rightarrow \omega_{AB} = 0$

$$\begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{Ax} \\ v_B^2 / r_{AC} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} d \\ r_{AC} \\ 0 \end{pmatrix} \rightarrow \quad a_A = \begin{pmatrix} -7 \\ 12 \end{pmatrix} \frac{ft}{s^2}$$

$$\alpha_{AB} = -3 \frac{rad}{s^2}$$

3)

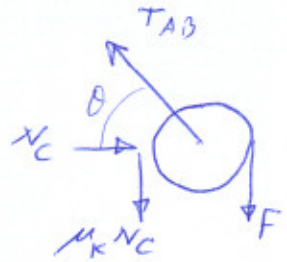


$$M = M_1 + M_2 \quad , \quad I_O = \frac{1}{12} M_1 (b^2 + c^2) + M_1 \left(a + \frac{c}{2}\right)^2 + \frac{1}{3} M_2 a^2$$

$$d = \frac{M_1 (a + c/2) + M_2 (\frac{a}{2})}{M}$$

$$\left.\begin{aligned} O_x &= M d \alpha \sin\theta + M d \omega^2 \cos\theta \\ Mg\, d \cos\theta &= I_O \alpha \\ O_y - Mg &= -M d \alpha \cos\theta + M d \omega^2 \sin\theta \end{aligned}\right\} \Rightarrow \begin{aligned} \alpha &= 13.65 \frac{rad}{sec^2} \\ O_x &= 54.8\ N \\ O_y &= 41.2\ N \end{aligned}$$

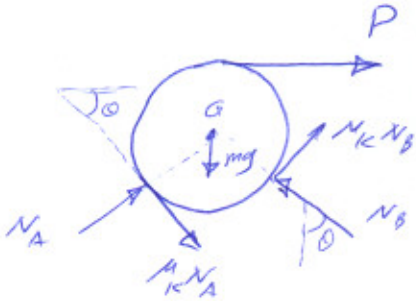
4)



$$\theta = \tan^{-1} \frac{a}{b}$$

$$\left.\begin{aligned} N_C - T_{AB} \cos\theta &= 0 \\ T_{AB} \sin\theta - \mu_K N_C - Mg - F &= 0 \\ F.b - \mu_K N_C b &= M k_A^2 \alpha \end{aligned}\right\} \Rightarrow \begin{aligned} N_C &= 102.818\ N \\ T_{AB} &= 267.327\ N \\ \alpha &= 7.281 \frac{rad}{s^2} \end{aligned}$$

5)



$$P - (N_B - N_A) \sin\theta + \mu_K (N_A + N_B) \cos\theta = 0$$

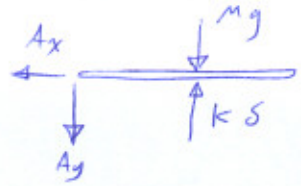
$$(N_A + N_B) \cos\theta + \mu_K (N_B - N_A) \sin\theta - mg = 0$$

$$[\mu_K (N_A + N_B) - P] r = \frac{1}{2} m r^2 \alpha$$

$$\Rightarrow N_A + N_B = \frac{mg - \mu_K P}{(1 + \mu_K^2) \cos\theta} \quad , \quad N_B - N_A = \frac{\mu_K mg + P}{(1 + \mu_K^2) \sin\theta}$$

$$, \quad \alpha = -\frac{2\mu_K}{mr} \cdot \frac{mg - \mu_K P}{(1 + \mu_K^2) \cos\theta} + \frac{2P}{mr}$$

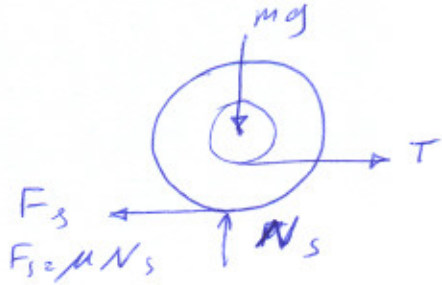
6) $-Mga + k\delta a = \frac{1}{3} M (a+b)^2 \alpha$



$A_x = 0$

$-A_y - Mg + k\delta = M\alpha \frac{a+b}{2}$ $\Rightarrow$ $\alpha = 23.1 \frac{rad}{s^2}$, $A_x = 0$, $A_y = 289 N$

7)



$T - F_s = m a_m$

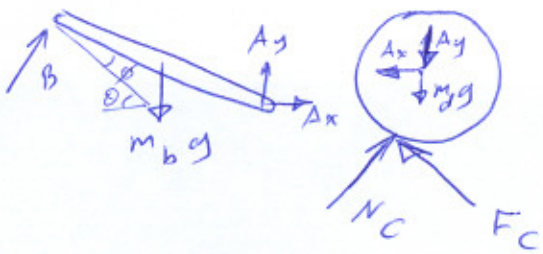
$N_s - mg = 0 \rightarrow N_s = mg$

$T r_i - F_s r_o = -m k^2 \alpha$

$F_{s,max} = \mu N_s$, $a_m = r_i \alpha$, $a_c = (r_o - r_i)\alpha$

$\Rightarrow$ $T = 2.32$ kN , $\alpha = 1.25 \frac{rad}{sec^2}$, $a_c = 1.348 \frac{m}{s^2}$

8) $\phi = \sin^{-1} \frac{r}{l}$



$N_C l \cos\phi - m_d g l \cos(\theta - \phi) - m_b g \frac{l}{2} \cos(\theta - \phi)$

$= \frac{-1}{2} m_d r^2 \alpha - m_d a_A r - m_b a_A r/2$

$-F_C + (m_D + m_b) g \sin\theta = (m_b + m_d) a_A$

$F_C r = 1/2\, m_d r^2 \alpha$

$a_A = r\alpha$ $F_{max} = \mu_s N_C$

$\rightarrow F_C > F_{max}$

$\Rightarrow$ $N_C = 67.966$ N

$F_C = 13.08$ N

$F_{max} = 10.195$ N

$\alpha = 10.9$ rad/s²

$\rightarrow F_C = \mu_K N_C$

$N_C = 67.966$ N

$F_C = 6.797$ N

$F_{max} = 10.195$ N

$\alpha = 5.664 \frac{rad}{s^2}$

9) $T_1 + U_1 = T_2 + U_2 \rightarrow 0 + 0 = 0 + 1/2\, k \left[\sqrt{(a+b)^2 + a^2} - b\right] - Mg \frac{a}{2}$

$\Rightarrow$ $k = 42.8$ N/m

10) $Mg\,d = \frac{1}{2} M v_G^2 + \frac{1}{2} I_G \omega^2 + Mg(d-r)$

$v_G = \omega\left(\frac{d}{2} - r\right) \rightarrow \omega = 12.4 \text{ , } v_G = 0.62$

$v_A = \omega \hat{k} \times \frac{d}{2}(-\hat{i}+\hat{j}) = -2.48(\hat{i}+\hat{j})\ m/s \rightarrow |v_A| = 3.5\ m/s$

11) $0 + 0 = \frac{1}{2} \frac{m_A r^2}{2} \left(\frac{v_b}{r}\right)^2 + \frac{1}{2}\left(\frac{m_B R^2}{2}\right)\left(\frac{v_b}{R}\right)^2 + \frac{1}{2} m_b v_b^2 - m_b g d$

$\rightarrow v_b = 1.519\ m/s$

12) انرژی کینتیک $\theta_0 = 0$ $\int_{\theta_0}^{\theta_0 + 90\,deg} -k\theta\, d\theta = \frac{1}{2} \cdot \frac{1}{3} M a^2 \omega^2$

$\Rightarrow \omega = \sqrt{\frac{3k\pi^2}{4Ma^2}}$

(روش دوم): $\frac{k}{2} \times \left(\frac{\pi}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{3} M a^2 \omega^2$

13) تا قبل از لغزش:

$m_b g r \sin\theta = (m_b r^2 + m_w k_G^2) \cdot \alpha$

$\rightarrow \alpha = \frac{m_b g r}{m_b r^2 + m_w k_G^2} \cdot \sin\theta$

$[\Sigma F_n = m a_n] \rightarrow F_N - m_b g \cos\theta = -m_b r \omega^2$

$[\Sigma F_t = m a_t] \rightarrow \mu_s F_N - m_b g \sin\theta = -m_b r \alpha$ لحظه لغزش:

$\Rightarrow \frac{\omega^2}{2} = \frac{m_b g r}{m_b r^2 + m_w k_G^2}(1 - \cos\theta)$

$\Rightarrow \alpha = 0.107 \frac{rad}{s^2}$, $\omega = 0.238 \frac{rad}{s}$, $\theta = 29.8\ deg$